[2][a] $y_2 = \boxed{v(x+1)}$ 1

$$y_2' = v(1) + v'(x+1)$$
$$= \boxed{v + v'(x+1)} \quad 1$$
$$y_2'' = v' + v'(1) + v''(x+1)$$
$$= \boxed{2v' + v''(x+1)} \quad 1$$

$$x(x+1)^2 y'' + (x+1)(x+3)y' - (x+3)y$$
$$= \boxed{\begin{array}{l} 2x(x+1)^2 v' + x(x+1)^3 v'' \\ \cancel{(x+1)(x+3)v} + (x+1)^2(x+3)v' \\ \cancel{-(x+1)(x+3)v} \end{array}} \quad 2\tfrac{1}{2}$$
$$= (x+1)^2[2x + (x+3)]v' + x(x+1)^3 v''$$
$$= (x+1)^2(3x+3)v' + x(x+1)^3 v''$$
$$= \boxed{3(x+1)^3 v' + x(x+1)^3 v'' = 0} \quad 1\tfrac{1}{2}$$

$\boxed{\text{CHECKPOINT: NO } v \text{ WITHOUT DERIVATIVE}}$ 1

LET $u = v'$, SO $u' = v''$

$$x\cancel{(x+1)^3}u' + 3\cancel{(x+1)^3}u = 0$$
$$\boxed{x\frac{du}{dx} + 3u = 0} \quad 1\tfrac{1}{2}$$
$$\frac{du}{dx} = \frac{-3u}{x}$$
$$\boxed{\int \frac{1}{u}\,du = \int -\frac{3}{x}\,dx} \quad \tfrac{1}{2}$$
$$\boxed{\ln|u| = -3\ln|x|} \quad \tfrac{1}{2}$$
$$v' = \boxed{u = x^{-3}} \quad \tfrac{1}{2}$$
$$\tfrac{1}{2} \quad \boxed{v = -\tfrac{1}{2}x^{-2} \text{ OR } x^{-2}}$$ EITHER v IS OK

$$\tfrac{1}{2} \quad \boxed{y_2 = x^{-2}(x+1) \text{ OR } x^{-1} + x^{-2}}$$ EITHER y_2 IS OK

[b] $y = \boxed{c_1(x+1) + c_2(x^{-1} + x^{-2})}$ 1

[c] $y(1) = \boxed{2c_1 + 2c_2 = 2}$ $\frac{1}{2}$ $\rightarrow c_1 + c_2 = 1$

$y' = \boxed{c_1(1) + c_2(-x^{-2} - 2x^{-3})}$ 1

$y'(1) = \boxed{c_1 - 3c_2 = 9}$ $\frac{1}{2}$

$c_1 + \quad c_2 = 1$

$-4c_2 = 8 \longrightarrow \boxed{\begin{array}{l} c_2 = -2 \\ c_1 = 1 - c_2 = 3 \end{array}}$ 1

$y = \boxed{3(x+1) - 2(x^{-1} + x^{-2})}$ 1

NOTE: ON MIDTERM, NEED TO SHOW WORK ON HOW YOU GOT THESE VALUES TO GET FULL CREDIT

[d] $a_n(x) = a_2(x) = \boxed{x(x+1)^2 = 0 \text{ @ } x = -1, 0}$ 1

LARGEST INTERVAL CONTAINING $x = -\frac{1}{2}$ WHERE $a_2(x) \neq 0$

IS $\boxed{(-1, 0)}$ 1

[3] $\boxed{4r^3-4r^2-15r+18=0}$ (1)

TRY $r=\pm\dfrac{1,2,3,6,9,18}{1,2,4}$

$=\pm 1,2,3,6,9,18,$
$\frac{1}{2},\ \frac{3}{2},\ \frac{9}{2},$
$\frac{1}{4},\ \frac{3}{4},\ \frac{9}{4}$

SIGN CHANGE   SIGN CHANGE

2 OR 0 POSITIVE ROOTS

$4(-r)^3-4(-r)^2-15(-r)+18$

$-4r^3-4r^2+15r+18$

SIGN CHANGE

1 NEGATIVE ROOT

TRY $r=-1$

$$
\begin{array}{r|rrrr}
-1 & 4 & -4 & -15 & 18 \\
 & & -4 & 8 & 7 \\
\hline
 & 4 & -8 & -7 & \boxed{25}
\end{array}
$$

NOT 0

$$
\boxed{
\begin{array}{r|rrrr}
-2 & 4 & -4 & -15 & 18 \\
 & & -8 & 24 & -18 \\
\hline
 & 4 & -12 & 9 & \boxed{0}
\end{array}
}
$$
(1½)

(½) $\boxed{(r+2)}\boxed{(4r^2-12r+9)}=0$ (½)

$(r+2)(2r-3)^2=0$

$r=\boxed{-2,\ \frac{3}{2},\ \frac{3}{2}}$ (1)

(1)

$y=\boxed{c_1e^{-2x}+c_2e^{\frac{3}{2}x}}+\boxed{c_3xe^{\frac{3}{2}x}}$

(1)

[4][a] $2r^2+(-1-2)r+3 = \boxed{2r^2-3r+3} = 0$ $1\frac{1}{2}$

$r = \boxed{\dfrac{3 \pm \sqrt{9-24}}{4} = \dfrac{3}{4} \pm \dfrac{\sqrt{15}}{4} i}$ 1

$y = \boxed{c_1 x^{\frac{3}{4}} \cos\left(\frac{\sqrt{15}}{4} \ln x\right) + c_2 x^{\frac{3}{4}} \sin\left(\frac{\sqrt{15}}{4} \ln x\right)}$ $1\frac{1}{2}$

[b] [i] $L[x^{\frac{1}{2}}] = \boxed{2x^2\left(-\frac{1}{4}x^{-\frac{3}{2}}\right) - x\left(\frac{1}{2}x^{-\frac{1}{2}}\right) + 3x^{\frac{1}{2}}}$ 1

$= -\frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}} + 3x^{\frac{1}{2}} = \boxed{2x^{\frac{1}{2}} \text{ OR } 2\sqrt{x}}$ $\frac{1}{2}$

[ii] $L[x^{-1}] = \boxed{2x^2(2x^{-3}) - x(-x^{-2}) + 3x^{-1}}$ 1

$= 4x^{-1} + x^{-1} + 3x^{-1} = \boxed{8x^{-1} \text{ OR } \frac{8}{x}}$ $\frac{1}{2}$

[c] $\dfrac{3}{x} - \dfrac{\sqrt{x}}{2} = \boxed{\dfrac{3}{8}\left(\dfrac{8}{x}\right) - \dfrac{1}{4}\left(2\sqrt{x}\right)}$ $1\frac{1}{2}$

$$y = \boxed{\frac{3}{8}x^{-1} - \frac{1}{4}x^{\frac{1}{2}} \text{ OR } \frac{3}{8x} - \frac{\sqrt{x}}{4}} \quad 1$$